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LYAPUNOV STABILITY ANALYSIS BASED ON AN IMPROVED OV MODEL

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ABSTRACT

The stability analysis of car-following model is an important content in the research process of car-following model. The stability analysis method based on an improved OV model was carried out. Different from the traditional linear stability analysis, the Lyapunov stability analysis of the improved OV car-following model was proposed from the perspective of control theory, and the theoretical derivation was given. The results show that, the stability analysis of the model using Lyapunov stability analysis method is consistent with Lyapunov theory. Therefore, the effectiveness of the Lyapunov stability analysis method is verified.

KEYWORDS: improved OV model, stability analysis, Lyapunov theory, car-following model.

1. INTRODUCTION

In recent years, with the rapid development of economy, traffic congestion, traffic safety and other related issues have become a hot social issue. In order to solve these problems, car-following model is the main method to study the normal and smooth operation of traffic flow, and has made a lot of progress.

In 1953, Pipes put forward the first car-following model, but the model only considered the relative velocity, but did not consider the influence of headway[1].In 1961, Newell proposed the Newell model considering the nonlinear effect of velocity[2]. In 1995, in order to solve the problem that the Newell model is not suitable for vehicle start-up and acceleration at signalized intersections, Bando proposed an optimized velocity model (OV model) by using the optimized velocity function[3]. In 1998, Helbing proposed the generalized force model (GF model), which solved the problem of high acceleration and unrealistic deceleration in OV model[4]. In 2006, Jiang Rui proposed the full velocity difference model (FVD model) on the basis of GF model and considering the influence of positive velocity difference[5]. On the basis of the FVD model, Wang Tao proposed a multiple velocity difference model (MFVD model) with better stability and considering the velocity difference information of several vehicles in front[6]. In 2012, Wang dianhai systematically reviewed the development process of car-following theory in the past 60 years, and divided car-following models into traffic engineering and physical statistics, which laid a solid foundation for further systematic study of car-following model[7]. Since then, with the rapid development of intelligent transportation system, it is particularly important to study the car-following model under the intelligent transportation system. This kind of model pays more attention to the "vehicle road coordination" and the relationship between the individual and the whole, analyzes the influence of car-following behavior on the traffic flow characteristics under the information environment, and provides the basic theory for microscopic traffic simulation and auxiliary driving control in complex environment[8]-[10].

In the above car-following model, OV model is a classical car-following model, but in this model, only the influence of vehicle spacing on car-following behavior is considered, and the influence of relative velocity on car-following behavior is not considered, which is inconsistent with the actual operation of vehicles. Therefore, this paper proposes an improved OV model. In order to effectively analyze the stability of improved OV model, this paper analyzes the stability of improved OV model based on Lyapunov function from the system point of view based on cybernetics[11].

2. ESTABLISHMENT OF IMPROVED OV MODEL

In 1995, Bando proposed an optimized velocity model, namely OV model, after introducing the optimized velocity function based on the preset safe velocity of drivers.

$$a_n(t)=\alpha[V(\Delta x_n(t))-v_n(t)] \quad (1)$$

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Where, $\Delta x_n(t)=x_{n+1}(t)-x_n(t)$ is the distance between two vehicles at time t. $x_n(t)$, $v_n(t)$ and $a_n(t)$ are the position, velocity and acceleration of the nth vehicle at time t. $V(\Delta x_n(t))$ is the optimal velocity function, namely,

$$V\left(\Delta x_n(t)\right)=\frac{V_{\max}}{2}\left[\tanh\left(\Delta x_n(t)-h_c\right)+\tanh\left(h_c\right)\right] \quad (1~~2~~)$$

Although the core idea of OV model is to optimize the optimal velocity of car-following with vehicle spacing, the influence of single factor of vehicle spacing on car-following behavior is not comprehensive enough. Therefore, after considering the distance and relative velocity of the two vehicles at the same time, the car-following model based on the improved optimized velocity is proposed as follow[11]

$$\frac{d^2x_n(t)}{dt^2}=k\left[V(\Delta x_n(t),\Delta v_n(t))-v_n(t)\right] \quad (2~~3~~)$$

Where, k is the driver's response sensitivity coefficient; $v_n(t)$ is the velocity of the nth vehicle at time t; $\Delta x_n(t)=x_{n+1}(t)-x_n(t)$ is the distance between two vehicles at time t; $V(\Delta x_n(t),\Delta v_n(t))$ is the improved optimized velocity function of the vehicle, namely,

$$V(\Delta x_n(t),\Delta v_n(t))=V\left(\Delta x_n(t)\right)+\alpha V\left(\Delta v_n(t)\right) \quad (3~~4~~)$$

Where, $V\left(\Delta v_n(t)\right)=V_f[\tanh(\Delta v_n(t)-v_s)+\tanh(v_s)]$, V_f is the ideal free flow velocity, v_s is the safe running velocity of the vehicle, h_c is the safe distance between vehicles.

3. STABILITY ANALYSIS

The stability analysis of traffic flow is an important problem in the study of car-following model. It is the basis of studying traffic congestion and traffic safety. It has important practical significance to improve road capacity and ease traffic congestion.

For the Lyapunov stability analysis of car-following model, we give the following corollaries and prove it theoretically:

When $k>0$, the improved OV model is Lyapunov stable.

The proof is as follows: In order to carry out stability analysis, according to Konishi's method[13-14], the improved OV model is rewritten as follows:

$$\begin{cases}\dfrac{d^2x_i(t)}{dt^2}=k[V(y_i(t))-\dfrac{dx_i(t)}{dt}]\\ y_i(t)=\Delta x_i(t)=x_{i+1}(t)-x_i(t) \\ \Delta v_i(t)=v_{i+1}(t)-v_i(t)\end{cases} \quad (4~~5~~)$$

Where, the above model can be further rewritten as the following formula:

$$\begin{cases}\dfrac{dv_i(t)}{dt}=k[V(y_i(t))-v_i(t)]\\ \dfrac{dy_i(t)}{dt}=v_{i+1}(t)-v_i(t)\end{cases} \quad (5~~6~~)$$

Assuming that the front of the car runs at a constant initial velocity, the steady state of the following vehicle is as follows:

$$\begin{bmatrix}v_i^*(t) & y_i^*(t)\end{bmatrix}^T=\begin{bmatrix}v_0 & V^{-1}(v_0)\end{bmatrix}^T \quad (6~~7~~)$$

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Therefore, by linearizing the system near the steady state, the following systems can be obtained:

$$\begin{cases} \dfrac{d\delta v_i(t)}{dt} = k[\Lambda \delta y_i(t) - \delta v_i(t)] \\ \dfrac{d\delta y_i(t)}{dt} = \delta v_{i+1}(t) - \delta v_i(t) \end{cases}$$ (7~~8~~)

Where,

$\delta v_i(t) = v_i(t) - v_0$, $\delta y_i(t) = y_i(t) - V^{-1}(v_0)$, Λ is the slope of OV function at $y_i(t) = V^{-1}(v_0)$, namely,

$$\Lambda = \frac{dV(y_i(t))}{dy_i(t)} \Big|_{y_i(t)=V^{-1}(v_0)}$$ (8~~9~~)

If $\delta v_i(t)$ and $\delta y_i(t)$ are selected as state variables, the state space expression of the system is

$$\begin{cases} \begin{bmatrix} \dfrac{d\delta v_i(t)}{dt} \\ \dfrac{d\delta y_i(t)}{dt} \end{bmatrix} = \begin{bmatrix} -k & k\Lambda \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \delta v_i(t) \\ \delta y_i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta v_{i+1}(t) \\ \delta v_i(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \delta v_i(t) \\ \delta y_i(t) \end{bmatrix} \end{cases}$$ (9~~10~~)

Let $P = \begin{bmatrix} \delta v_i(t) & \delta y_i(t) \end{bmatrix}^T$, $A = \begin{bmatrix} -k & k\Lambda \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$

Then the equation is expressed as

$$\begin{cases} \dot{P} = AP + B\delta v_{i+1}(t) \\ \delta v_i(t) = CP \end{cases}$$ (10~~11~~)

Therefore, the following Lyapunov functions are selected

$$P(x) = \delta v_i(t)^2 + k\Lambda \delta y_i(t)^2$$ (11~~12~~)

Then,

$$\dot{P}(x) = 2\delta v_i(t)[(-k)\delta v_i(t) + k\Lambda \delta y_i(t)] + 2k\Lambda \delta y_i(t)(-\delta y_i(t)) = -2k\delta v_i(t)^2$$ (12~~13~~)

When $k > 0$, for $\dot{P}(x) < 0$, so the improved OV model is Lyapunov stable.

4. CONCLUSION

OV model is a classic model of traffic flow theory, but the influence of relative velocity on car-following behavior is not considered in this model, and the stability research method is relatively simple. In this paper, an improved OV model is established and its stability is analyzed. Different from the traditional linear analysis method, the Lyapunov analysis of the car-following model is carried out from the perspective of control theory, and the strict theoretical derivation is given. Finally, the stability of the model conforms to the Lyapunov theory.

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